

Numerical Simulation of the Dynamics of Hepatitis B Infection with Infective Travelers using Immunization and Treatment as Control

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Abstract

Background: Contamination with Hepatitis B infection (HBV) is a challenge to worldwide wellbeing generally because of increased movement. **Aim:** The aim of this study is to model the impact of relapse in patients with chronic Hepatitis b virus infection, using immunization and treatment strategies as control. **Method:** In this exploration, we take on the model of Nwaokolo et al. 2020 which is an extension of a six deterministic compartmental model that has been established with the point of foreigners being exposed to test to reduce the amount of travelers with HBV (Khan et al. 2016). **Results:** Consequently, to reflect reality, we simulate the model to concentrate on the effect of immunization and treatment on the dynamics of HBV with infective travelers. The sensitivity examination and mathematical simulations reveals that rising immunization rate has high effect than treatment rate in reducing the effective reproduction number which thusly diminishes HBV transmission in the populace. Likewise, numerical outcomes show that adherence to high-effective immunization and treatment strategies are expected to limit the spread of HBV with infective travelers.

Keywords: Hepatitis B Virus, travelers, immunization, treatment

1. Introduction

Hepatitis B Infection (HBV) is a major cause of mortality around the world (Visvanathan et al., 2016). The spread of HBV influence the demand for medical care services for the most part because of increased movement from nations with intermediate/high HBV predominance (Shadur et al., 2013). The prevalence of HBV infection varies all through regions of the world (Trepo et al., 2014) and could be sorted as high (>8%) in China and Sub-Saharan African Nations like Nigeria (Ani et al., 2020; Adeoye, 2010), modestly endemic (2-8%) in Eastern and southern Europe, and low (0.5-2%) in Northern and Western Europe (Trepo et al., 2014). At present, there are roughly 2 billion individuals overall that have been contaminated with HBV, representing over 350 million cases of chronic active hepatitis (EASL, 2013) and three-quarter of them are from China. Without mediation, 887,000 will ultimately pass on from HBV disease each year (WHO, 2019). HBV is spread by birth (from mother to youngster), sex and trade of blood and body liquids. Be that as it may, the spread could turn out to be quicker through non-standard condition and structure of migration process. Hence, as the disease spreads the link between migration and the spread of HBV is arising stronger. Subsequently, movement appears to comprise an issue in the process of studying the disease. A few studies uncover

the viability of screening of travelers born in nations with prevalence of HBV infection of 2% or greater, in order to deport those who are infected with HBV (Ahmed and Foster, 2010). Thus, the deportation of infected travelers could prompt criticism and HBV-related deaths. Besides, infected travelers could become distant to the general public health administration yet at the same time keep in touch with the susceptible populace.

In this way, world health organization (WHO) unequivocally suggests that health checks be offered, to guarantee admittance to health for all transients, requiring health security, while sticking to the rule of basic freedoms and pride of travelers. Subsequently, prevention of transmission of HBV between infective travelers and the occupant populace seems, by all accounts, to be a significant stage in decreasing the worldwide weight of HBV disease. Thus, with an end goal to control HBV movement within the infective travelers and resident populace, immunization and treatment would be required. HBV control measures include immunization, education, screening of endlessly blood items and treatment (CDC, 2013). To understand the dynamics of HBV disease, a few numerical models have been formed (Khan et al., 2016; Nwaokolo et al., 2020).

This study applies the model of Nwaokolo et al., 2020 which is an extension of the work by Khan et al. (2016). The exploration recommends that travelers for short visit and students ought to be exposed to test to lessen the quantity of travelers with infection. The model (Nwaokolo et al., 2020) has proactively been studied for numerical and epidemiological well-posedness, and global stability analysis. Subsequently, the current review plans to lead a conduct a sensitivity analysis and numerical simulation of the effect of immunization and treatment on the dynamics of HBV within the sight of infective travelers, as suggested.

2. Related Works

Several researchers have used mathematical models to study the transmission process and treatment of the hepatitis B virus as well as other types of hepatitis virus, and the impact of preemptive measures and strategies on its spread. O'Leary et al. (2009) constructed mathematical models of HBV immunization and drug treatment by comparing the effects of treatment and vaccination. Jianhua et al. (2010) developed a model to evaluate the impact of vaccination and other forms of HBV infection control measures. Qiang et al. (2014) developed a dynamic model with seven cell forms centered on the biological mechanisms of viral repetition and the host immune response, and this model projected that appropriate long-term therapy was necessary to decrease the symptoms of HBV and to sustain treatment benefits. Demasse et al. (2016) developed a model of the age structure of HBV transmission that contains asymptomatic and symptomatic infections. Liu et al. (2017) used a SIRS infectious disease model with constant vaccination and age structure. Tao et al. (2017) employed a model of hepatitis B virus transmission considering a period of infection when incidence transmission was nonlinear taking into account vertical transmission. Zhao et al. (2017) established a kinetic model for hepatitis B and analyzed the data of new cases of hepatitis B in Xinjiang from 2005 to 2014. Su et al. (2018) developed a dynamic model for hepatitis B virus transmission with mechanisms of vertical transmission and vaccination. Sun et al. (2020) evaluated the influence of physical immunization and examination on hepatitis B. Li et al. (2021) developed a mathematical model for the pharmacological barrier of HBV transmission between mothers and infants throughout maternal pregnancy period. Habenom et al. (2021) developed a fractional order mathematical model of hepatitis B virus transmission and studied possible strategies for hepatitis B vaccination and isolation control. Goyal et al. (2021) established novel mathematical models that integrated these key biological progressions and examined them

both analytically and numerically, and the analysis recommended that existence of some form of a selective advantage of infected hepatocytes containing only IDNA to explain the viral dynamics observed during antiviral treatment and the transition from peak to acute infection.

Asfaw et al. (2022) developed a dynamic model which demonstrated that the solution of the dynamical system under consideration is positive and bounded. The effective reproductive number that represents the epidemic indicator is generated from the biggest eigenvalue of the net generation matrix, and the model is examined qualitatively using differential equation stability theory. For disease free and endemic equilibria, both local and global asymptotic stability criteria are determined. According their findings shows that, vaccination and treatment interventions play a critical rola in reducing HBV reproduction and transmission. Peijiang et al. (2022) developed a fractional model for the transmission dynamics of Hepatitis B. They investigated the transmission due to different infection phases of the Hepatitis B virus and constructed a nonlinear epidemic. In their study, the fractional hepatitis B virus model with Atangana–Baleanu derivative (AB derivative) is formulated with vaccine effects. Xu et al. (2023) developed a dynamic model which included a compartment for potentially infectious hepatitis B virus infections. Their results reveal that the number of potential infectious hepatitis B virus infections is so high that the number of hepatitis B patients persistently rises in China.

In this study, we conducted numerical simulation to evaluate the impact of relapse in patients with chronic Hepatitis b virus infection, using immunization and treatment strategies as control. MATLAB was used to analyze changes in the number of incidences, thereby exploring effective strategies for controlling hepatitis B virus transmission immunization and treatment for travelers.

3. Methodology

3.1 Mathematical Model

The HBV model by Nwaokolo *et al.* (2020) with state variables (Table 1) and parameters (Table 2) is governed by the following variable and parameters.

Table 1: Variables and Parameters of the Model

Parameters	Interpretation
S (t)	Number of Susceptible persons at time t.
E (t)	Number of Exposed persons at time t.
A (t)	Number of Intense infective at time t.
C (t)	Number of Chronic carriers at time t.
V (t)	Number of immunized persons at time t
M (t)	Number of migrated persons at time t.
T (t)	Number of treated persons at time t
R (t)	Number of recuperated persons at time t
δ	Equal per capita birth and demise rate
π	The proportion without immunization
γ_1	The rate at which exposed persons become infectious and move to the intensely infected class.
γ_2	The rate at which intensely infected persons move to the chronic carrier class.
γ_3	The rate at which chronic carriers secure immunity and move to the vaccinated class.
β	The transmission coefficient
κ	The irresistibility of carriers comparative with intense infections.
q	The proportion of intensely infected persons that become chronic disease carriers.
1 - q	The proportion of intensely infected persons that move to the immunity class.
δ_0	The loss of immunity from the immunized class to susceptible class.
α_0	The ratio of chronic carriers that are treated per unit time.
φ	The rate at which treated persons relapse and continue to the chronic class.
ρ	The proportion of immunized susceptible per unit time.
ξ	Rate of flow from exposed class to migrated class.
α	The flow from migrated class to susceptible class.
μ_1	The transmission rate from migrated class to exposed class.
μ_2	The transmission rate from migrated class to intensely infective class.
η	The proportion of the immunized kids born by carrier mothers.
$\delta(1 - \pi)$	The babies that are effectively immunized.
$\delta\pi(1 - \eta C(t))$	Birth transition into the susceptible class.

The extended model equation is given below;

$$\frac{dS}{dt} = \delta\pi(1 - \eta C) - \delta S - \beta(A + kC)S + \delta_0 V - pS + \alpha M, \quad (1)$$

$$\frac{dE}{dt} = \beta(A + kC)S - \delta E + \delta\pi\eta C - \gamma_1 E - \xi E + \mu_1 M, \quad (2)$$

$$\frac{dA}{dt} = \gamma_1 E - (\delta + \gamma_2)A + \mu_2 M, \quad (3)$$

$$\frac{dC}{dt} = q\gamma_2 A + \varphi T - (\delta + \alpha_0)C, \quad (4)$$

$$\frac{dT}{dt} = \alpha_0 C - (\delta + \varphi + \gamma_3)T, \quad (5)$$

$$\frac{dM}{dt} = \xi E - (\mu_1 + \mu_2 + \delta + \alpha)M, \quad (6)$$

$$\frac{dV}{dt} = \delta(1 - \pi) + pS - (\delta + \delta_0)V. \quad (7)$$

Where,

$$R = 1 - S - E - A - C - T - M - V \tag{8}$$

And,

$$S(0) > 0, E(0) \geq 0, A(0) \geq 0, C(0) \geq 0, T(0) \geq 0, M(0) \geq 0, V(0) \geq 0$$

In this way, the laid out reproduction number of the model (1-7) as contained in Nwaokolo et al (2020) is

$$R_{r^c} = \frac{[\beta S^0(c + Kq\gamma_2(\delta + \varphi + \gamma_3)) + e(\delta + \varphi + \gamma_3)](\mu_2\xi + \gamma_1a) + \xi\mu_1bc}{dbca} \tag{9}$$

Where,

$$S^0 = \frac{\delta\pi + \delta_0}{\delta + \delta_0 + p}, a = (\alpha + \delta + \mu_1 + \mu_2), b = (\delta + \gamma_2), c = (\delta + \gamma_3)(\delta + \alpha_0) + \delta\varphi,$$

$$d = (\delta + \xi + \gamma_1), e = \delta\pi\eta q\gamma_2, f = (\delta + \varphi + \gamma_3), g = (\mu_2\xi + \gamma_1a)$$

In any case, we are keen on looking at in the event that further research might pass on astonishing new data in controlling the spread of hepatitis B infection with infective travelers. Subsequently, spurred by the work of Nwaokolo et al. (2020) the current work expects to study the sensitivity analysis of the effect of immunization and treatment on HBV transmission with infective travelers.

Consequently, $R_{r^c} < 1$, actually intends that on a average, an HBV infected individual will produce less than one HBV infection.

Thus, $R_{r^c} < 1$, infers

$$\beta_r < \frac{dbca - (e(\mu_2\xi + \gamma_1a)(\delta + \varphi + \gamma_3) + \xi\mu_1bc)}{S^0((c + Kq\gamma_2(\delta + \varphi + \gamma_3)) + e(\delta + \varphi + \gamma_3))(\mu_2\xi + \gamma_1a) + \xi\mu_1bc} \tag{10}$$

where

$$S^0 = \frac{\delta\pi + \delta_0}{\delta + \delta_0 + p}$$

Subsequently, for HBV to be put under control the transmission rate shouldn't surpass the upper bound of equation (10).

The effective reproduction number without relapse ($\varphi=0$) consequently becomes

$$R_{e^c} = \frac{(\beta S^0((\delta + \alpha_0) + Kq\gamma_2) + e)(\mu_2\xi + \gamma_1a) + \xi\mu_1b(\delta + \alpha_0)}{dba(\delta + \alpha_0)} \tag{11}$$

4. Results and Discussion

4.1 Numerical Simulation

Numerical Simulations is utilized to outline some of the analytical outcome and check theoretical expectations of the model (1-7) utilizing set of parameter values (see Table 2), coded by the guide of MATLAB program to recreate the model.

Table 2: Parameters Values used in Numerical Simulation

<i>Parameters</i>	<i>Range</i>	<i>Value</i>	<i>Source</i>
β		0.8	Khan <i>et al.</i> (2016)
δ		0.0143	Khan <i>et al.</i> (2016)
δ_o	0.03-0.06	0.03	Khan <i>et al.</i> (2016)
γ_1		6 per year	Khan <i>et al.</i> (2016)
γ_2		4 per year	Khan <i>et al.</i> (2016)
γ_3		0.34	Khan <i>et al.</i> (2016)
k		0.1	Khan <i>et al.</i> (2016)
π		0.8	Khan <i>et al.</i> (2016)
η		0.7	Khan <i>et al.</i> (2016)
μ_1		0.1	Khan <i>et al.</i> (2016)
μ_2		0.1	Khan <i>et al.</i> (2016)
α	0-1	0.1	Khan <i>et al.</i> (2016)
ξ	0-1	0.1	Khan <i>et al.</i> (2016)
q	0.05-0.9	0.05	WHO (2002)
ϕ		0.393	Pan <i>et al.</i> .(2013)
S_o		0.493	Medley <i>et al.</i> .(2001)
E_o		0.0035	Medley <i>et al.</i> .(2001)
A_o		0.0035	Medley <i>et al.</i> .(2001)
C_o		0.25	Assumed
M_o		0.003	Assumed
P		0.8	Assumed
α_o		0.8	Assumed

Consequently, utilizing the study of disease transmission of HBV gotten from literature (Khan et al., 2016; Medley et al., 2001), the results of the numerical experiments are intended to study the following cases.

4.2 Numerical Experiments

The behaviour of the model is shown in Figure 1 - 12 where we plot the prevalence of the population with time. The parameter values used in the numerical experiment is in Table 2. We present the numerical results of the experiments in this section as below;

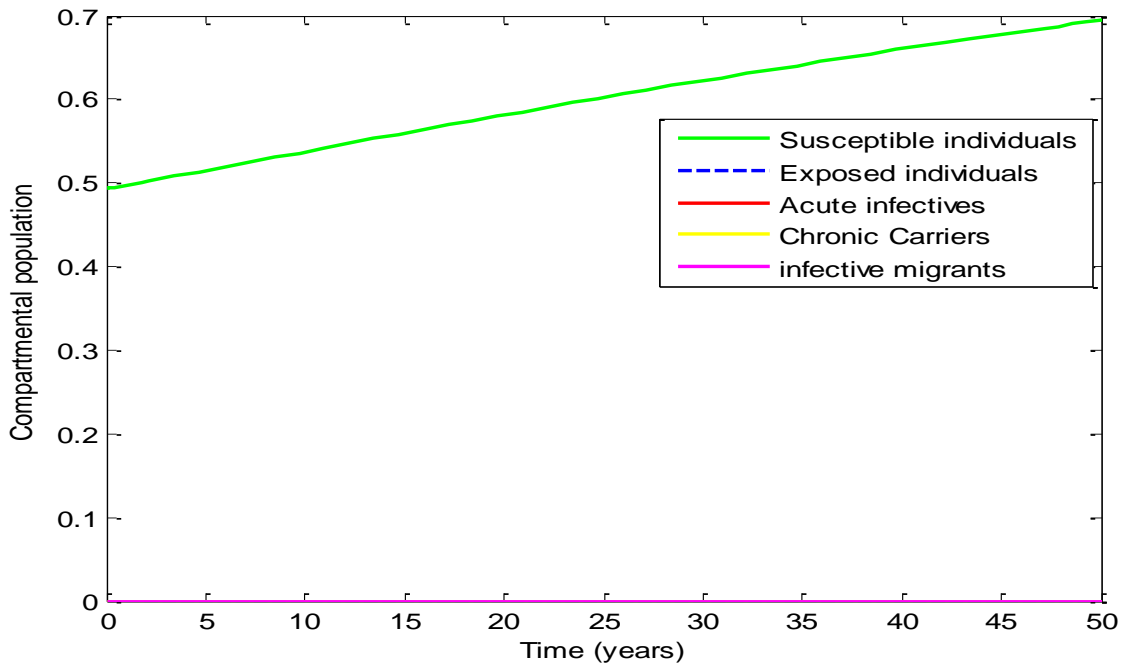


Figure 1: Prevalence Rate in a Disease-Free State

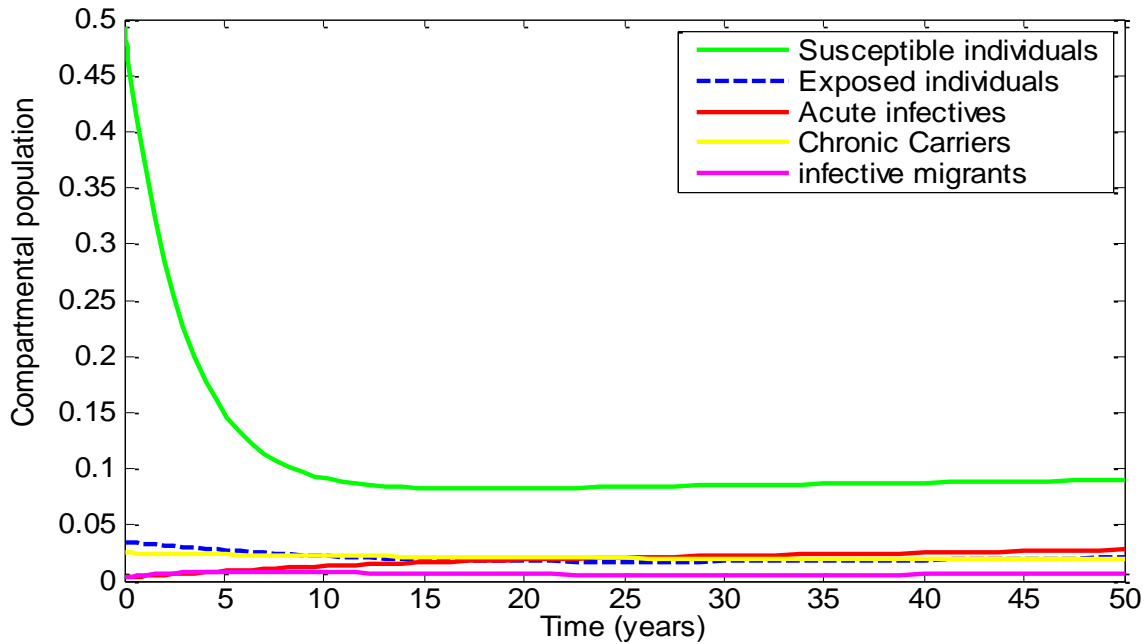


Figure 2: Compartmental Populace Showing What is going on where there is low Immunization Rate with no treatment in Controlling the Spread of HBV with Infective Travelers ($p = 30\%$, $\alpha_0 = 0\%$)

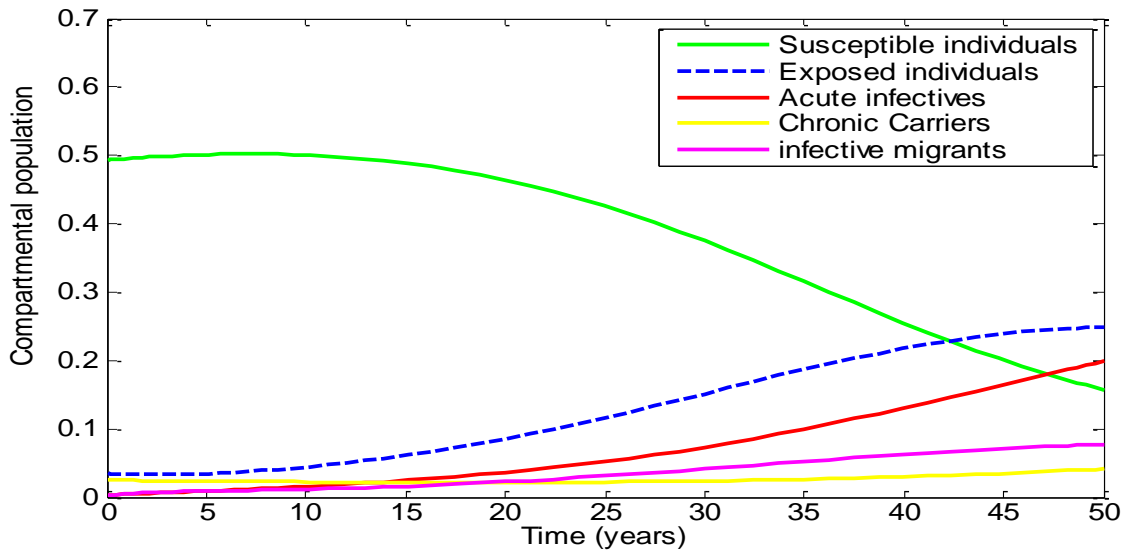


Figure 3: Compartmental Populace Showing What is happening where there is neither Immunization nor Treatment Rate in Controlling the Spread of HBV with Infective Travelers ($p = 0\%$, $\alpha_0 = 0\%$)

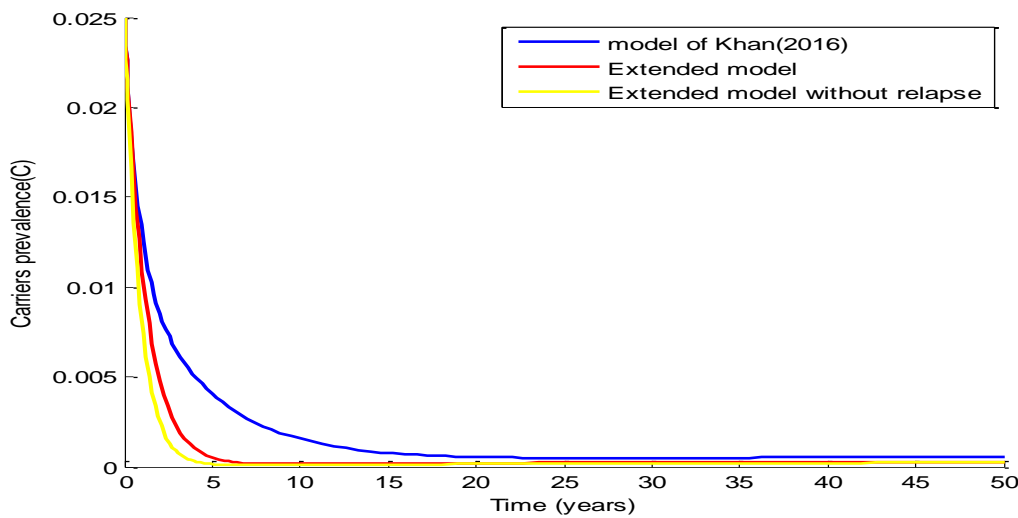


Figure 4: Population Conduct of chronic carriers in a Circumstance where (I) there is no Treatment except for Low immunization Rate in Controlling the Spread of HBV with Infective Travelers (Khan et al., 2016) ($p = 30\%$, $\alpha_0 = 0$) (ii) there is Low immunization rate and Treatment with Relapse on carriers prevalence with infective travelers (Extended model) ($p = 30\%$, $\alpha_0 = 30\%$, $\varphi = 0.393$) (iii) there is Low immunization rate and Treatment without Relapse on carriers prevalence with infective travelers (Extended model without relapse) ($p = 30\%$, $\alpha_0 = 30\%$, $\varphi = 0$)

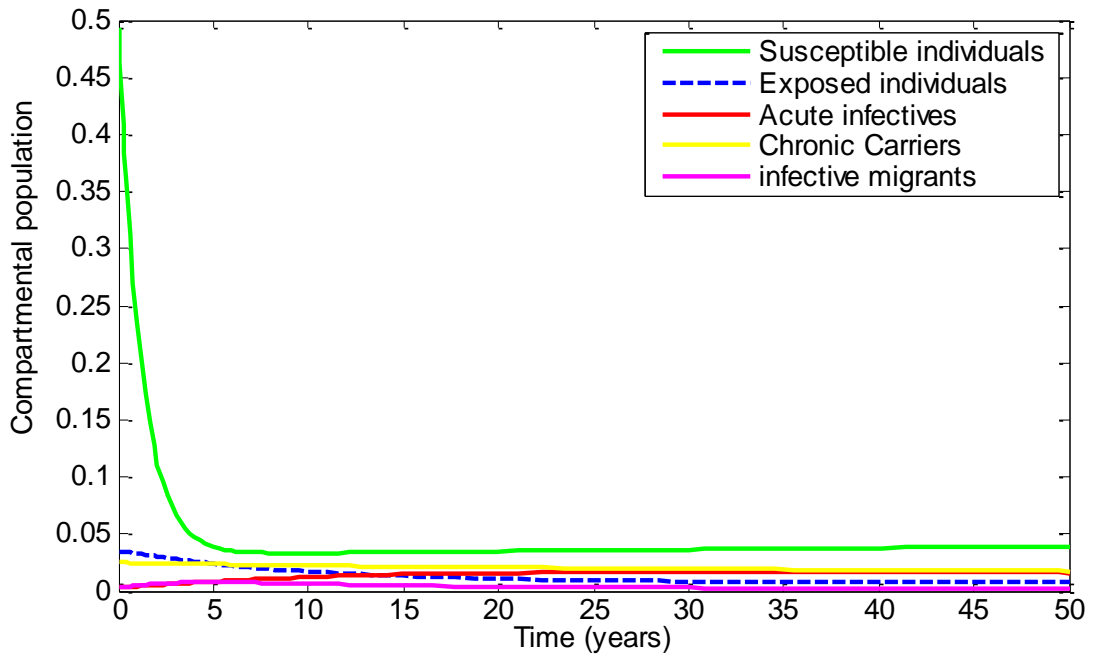


Figure 5: Compartmental Populace Showing What is happening where there is no Treatment except for high Immunization Rate in Controlling the Spread of HBV with Infective Travelers. ($p = 80\%$, $\alpha_0 = 0\%$)

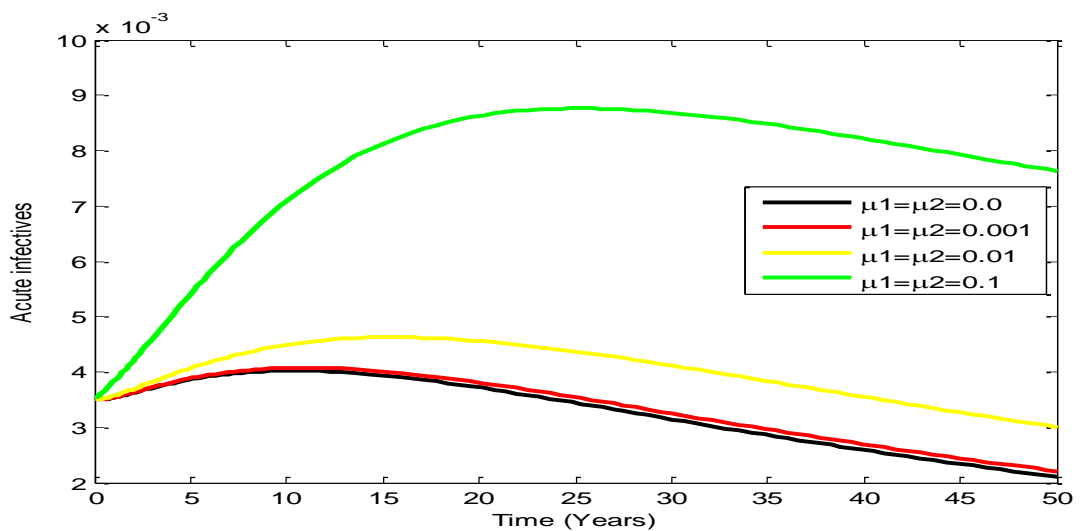


Figure 6: Effect of Various Transmission rates of Infective Travelers on Intense infective People

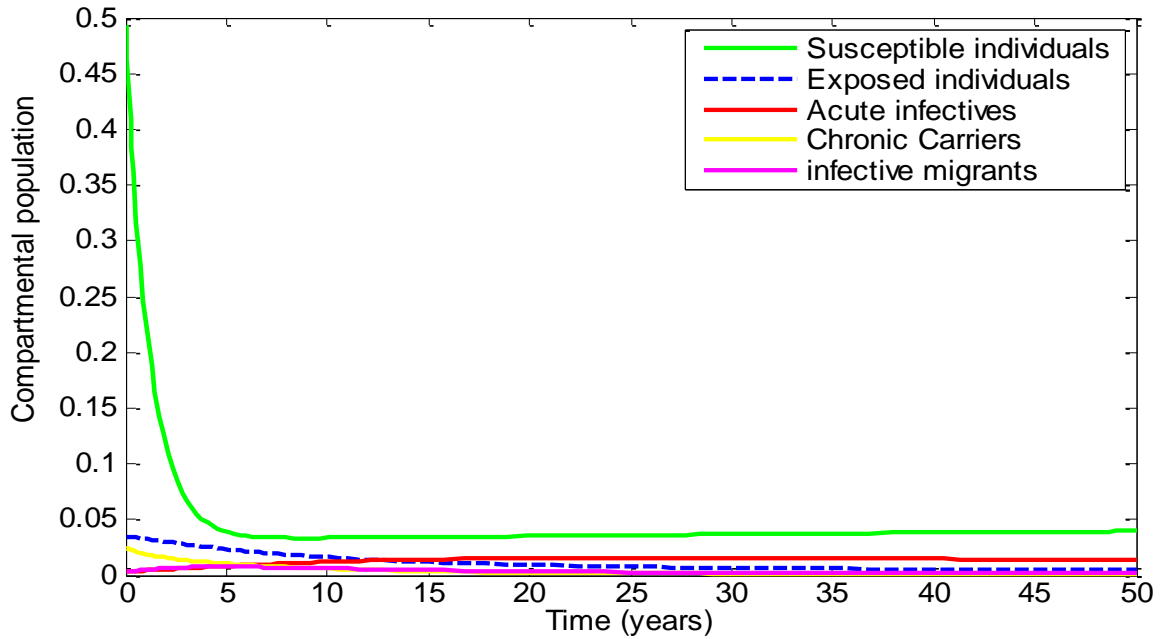


Figure 7: Compartmental Populace Showing What is going on where there is High Immunization and Low Treatment Rate in Controlling the Spread of HBV with Infective Traveler($p = 80\%$, $\alpha_0 = 30\%$)

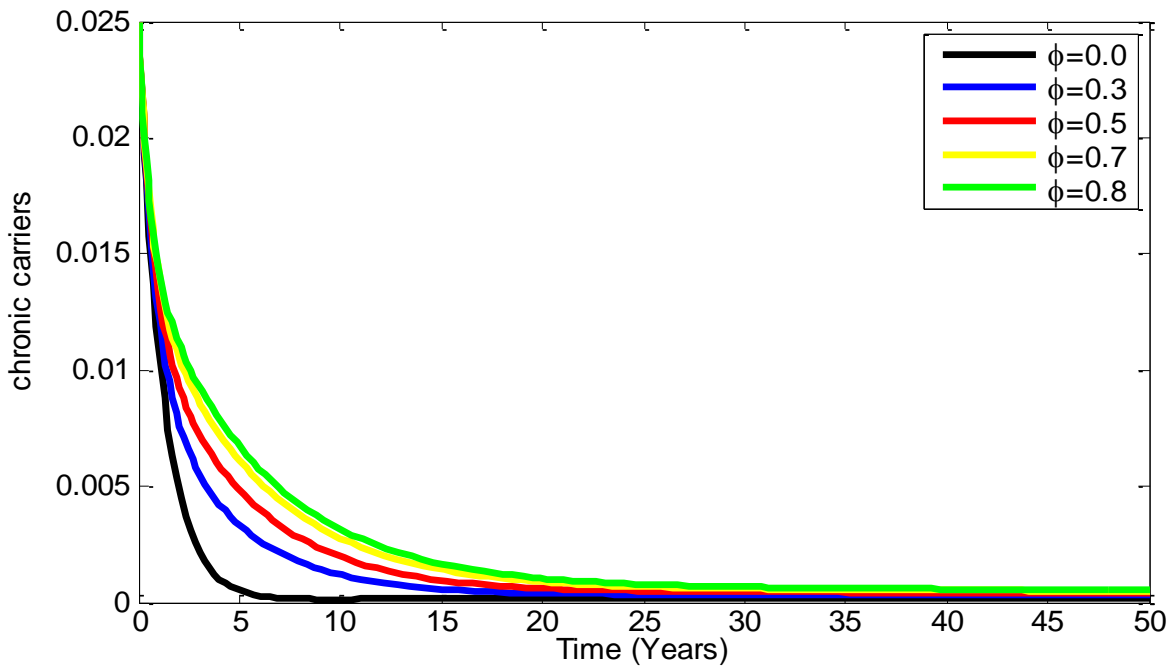


Figure 8: Compartmental Populace Showing What is going on where there is High Immunization and Treatment Rate in Controlling the Spread of HBV with Infective Traveler($p = 80\%$, $\alpha_0 = 80\%$)

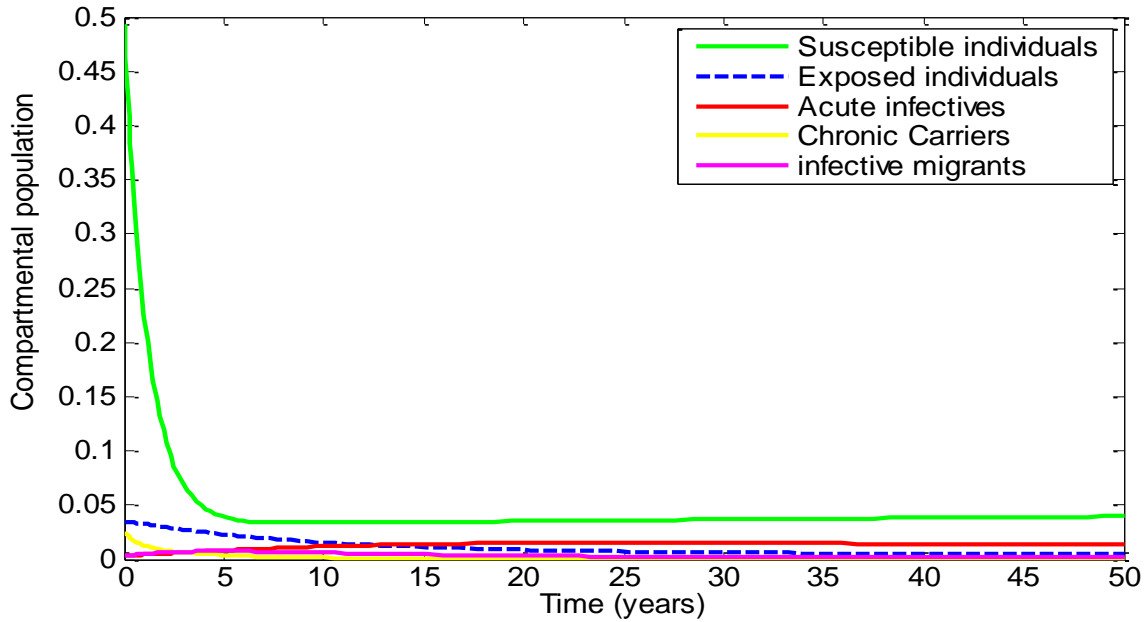


Figure 9: Compartmental Populace Showing What is going on where there is High Immunization and Low Treatment Rate in Controlling the Spread of HBV with Infective Traveler ($p = 80\%$, $\alpha_0 = 30\%$)

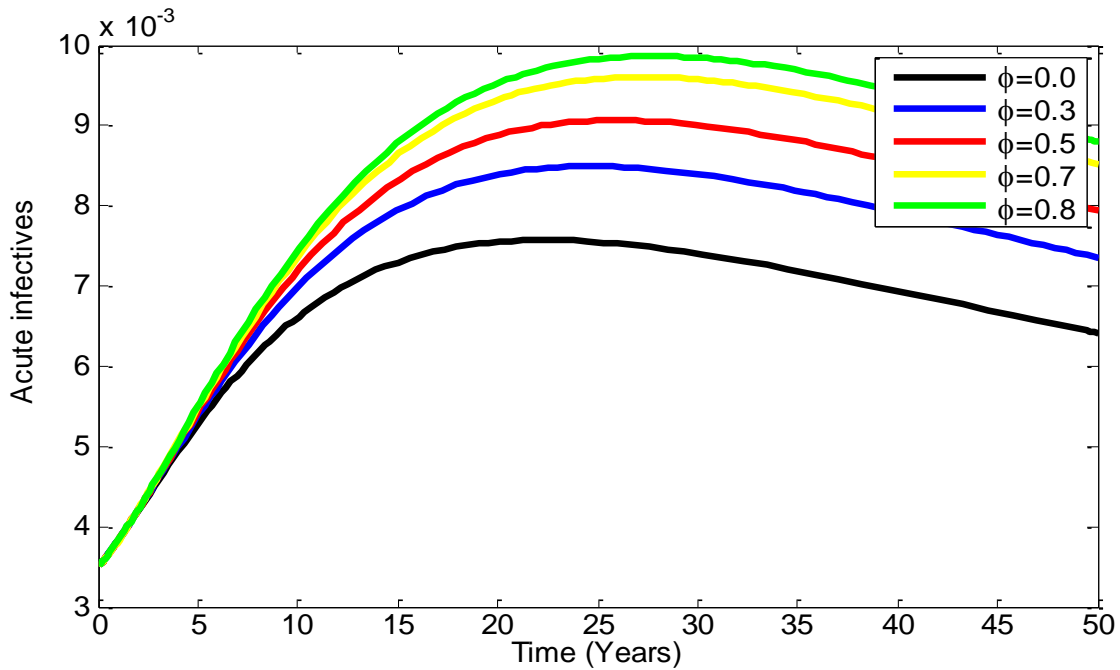


Figure 10: Effect of Relapse Rates on Intense Infective in the Presence of Infective Travelers ($\varphi = 0, 0.3, 0.5, 0.7, 0.8$)

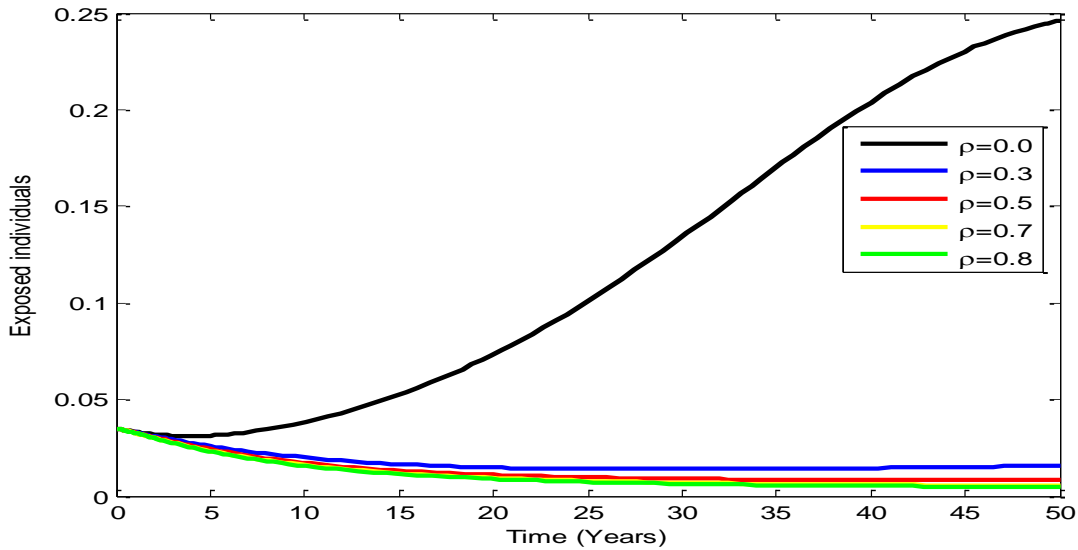


Figure 11: The Effect of Changed Immunization Rate on Susceptible People ($\rho = 0\%, 30\%, 50\%, 70\%, 80\%$)

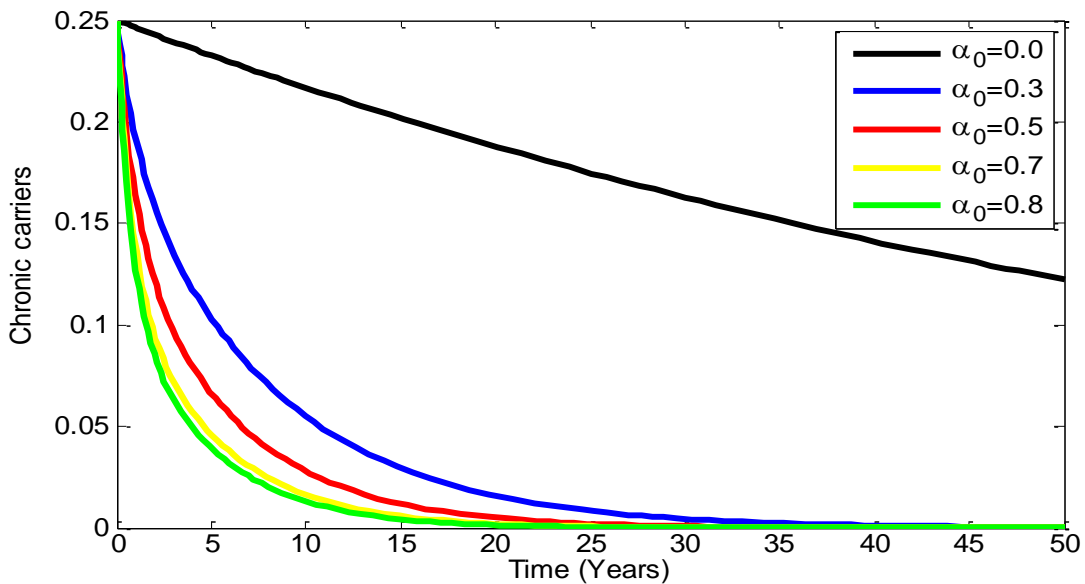


Figure 12: Effect of Fluctuated Treatment Rate on Persistent carriers in the presence of Infective Travelers ($\alpha_0 = 0\%, 30\%, 50\%, 70\%, 80\%$)

Figure 1 clearly indicates that there is no disease (HBV) in the population. The medical implication is that the Hepatitis B surface antigen and antibodies are all negative in the population (that is, there is absence of hepatitis B surface antigen and antibodies). Figure 2 indicates that with 30% vaccination rate there is a slight decrease in exposed, acute and chronic carriers. This simply means that the infection remains in the population because vaccination does not have direct impact on them. Biologically, the finding is similar to the report of Weinbaum *et al.* (2008) that, those susceptible that are vaccinated have

acquired immunity or produce effective immune response. This occurs when the hepatitis B surface antigen is negative and the antibody is positive. Hence, since there are many susceptible individuals that are not yet vaccinated, the number of susceptible-exposed to infection is high as shown in Figure 2.

Figure 3 indicates that when there is neither vaccination nor treatment, the exposed, acutely infected and chronic carriers' increases rapidly over a long period of time. Medically, the result in Figure 3 is in line with the report of Liang (2009) that the rapid spread is due to the presence of Hepatitis B envelope antigen (HBeAg), along with high viral replication and infectivity. Figure 4 shows that with 30% vaccination without treatment rate it takes the longest time for chronic carrier die out in the population (Khan *et al.*, 2016). However, when 30% vaccination and treatment rate with relapse (0.393) in the extended model are employed concurrently, the chronic carrier dies out faster compared to a single intervention (extended model). Furthermore, with 30% vaccination and treatment rate on carriers takes a short time or proves best when this three graph are viewed together(extended model without relapse).

Figure 5 shows that with 80% vaccination rate there is a decrease in acute, exposed and chronic carriers. This is because vaccination helps to reduce the risk of new infection. The biological interpretation of the result in Figure 5 is consistent with the report of Weinbaum *et al.* (2008) that, those susceptible that are vaccinated have acquired immunity or produce effective immune response. Hence, since there are few susceptible to be infected, the number of susceptible-exposed to infection is low as compared to Figure 2. Figure 6 illustrates that with an increase in vaccination rate (0%, 30%, 50%, 70%, 80%), the number exposed to infection is remarkably reduced within a short period if susceptible are vaccinated at higher rates, in other to prevent large spread of infection in the population. Figure 7 shows that with 80% vaccination and 30% treatment rate there is a decrease in acute and exposed individuals. Whereas, the chronic carriers dies off in about 20 years as compared to Figure 5. This is because the combined impact of vaccination and treatment decreases chronic HBV infection than a single intervention strategy. Medically, the result in Figure 7 is in line with the report of Liang (2009) and Weinbaum *et al.* (2008) that HBeAg will clear over time through treatment which leads to reduction in disease activity, viral load and histological improvement over time in the population.

Figure 8 shows clearly that with 80% vaccination and treatment rates, there is remarkable decrease in the number of new chronic infection, which dies out in about 8-12 years as compared to Figure 7. The medical implication is that the result in Figure 8 agrees with the report of Weinbaum *et al.* (2008) that HBeAg will clear over time through treatment which leads to marked reduction in disease activity, viral load and histological improvement over time in the population. This agrees with the requirement for global elimination of hepatitis B (Nayagam *et al.*, 2016). Figure 9 shows that with 80% vaccination and 30% treatment rate there is a decrease in acute and exposed individuals. Whereas, the chronic carriers dies off in about 20 years as compared to Figure 11. This is because the combined impact of vaccination and treatment decreases chronic HBV infection than a single intervention strategy. Medically, the result in Figure 9 is in line with the report of Liang (2009) and Weinbaum *et al.* (2008) that HBeAg will clear over time through treatment which leads to reduction in disease activity, viral load and histological improvement over time in the population. Figure 10 demonstrates clearly that when the relapse rate is increased (0%, 30%, 50%, 70%, 80%), it takes a longer time for chronic (carrier) infection to be curtailed. On the other hand, when relapse rate is reduced by adhering to treatment it takes a short time for the infection to be curtailed. This agrees with the work by Nwaokolo *et al.* (2022) that relapse increases the number of infected

persons and thus the spread of the Hepatitis B Virus. Also, it delays the quick response to the treatment regimen which consequently slows down the potential of recovery from the infection. Therefore, for the infection to be put under control, either short or long term depend on the patients adherence to treatment.

Figure 11 illustrates that with an increase in vaccination rate (0%, 30%, 50%, 70%, 80%), the number exposed to infection is remarkably reduced within a short period if susceptible are vaccinated at higher rates, in other to prevent large spread of infection in the population. Figure 12 reveals that with an increase in transmission rate (0, 0.001, 0.01, 0.1) of infective migrants, the acute infective individuals increases drastically, reaches a peak and then begins to decline but not to zero, over a long period of time. This is because at this threshold the infective population becomes stable due to acquired (natural) immunity or saturation point.

4.3 Sensitivity Analysis

The normalized forward sensitivity index of a variable τ , which depends differentiable on index with respect to the parameter p is defined as

$$r_{\tau}^{\rho} = \frac{\partial \rho}{\partial \tau} \times \frac{\tau}{\rho} \tag{12}$$

From the formula of effective reproduction, R_{τ}^c in equation (9), we derive analytical expression for the sensitivity of R_{τ}^c as

$$r_{\tau}^{R_{\tau}^c} = \frac{\partial R_{\tau}^c}{\partial \tau} \times \frac{\tau}{R_{\tau}^c} \tag{13}$$

where τ denotes the parameter. We compute the sensitive indices of the model systems (1-7) for some parameter involved in R_{τ}^c . We use the parameter values displayed in the Table 3 to obtain the numerical value, therefore, the sensitivity indices results of R_{τ}^c are given in Table 4 and are arranged from the highest sensitivity value to the lowest value. The indices with positive signs show that the value of R_{τ}^c increases when the corresponding parameters are increased and those having negative signs indicate that the value of R_{τ}^c decreases when the parameters are increased.

Table 3: Sensitivity Indices of R_r^c with respect to some Parameters

S/N	Parameter	Sensitivity Index	Sign
1	β	1.000000000	+
2	p	0.9475304984	-
3	δ_0	0.6884058306	+
4	γ_2	0.434756444	-
5	μ_2	0.3475541027	+
6	α	0.3123292643	-
7	π	0.2760833440	+
8	γ_1	0.1742824945	+
9	ξ	0.0296644089	-
10	μ_1	0.0094382464	+
11	q	0.00003554198	+
12	α_0	0.00003425081	-
13	η	0.00002156809	+
14	φ	0.00001801218	+
15	k	0.0000139739	+
16	γ_3	0.000001728521	-

In interpreting the sensitivity indices, we keep all other factors constant. Table 3 shows that parameters $\beta, \delta_0, \pi, q, \gamma_1$ and η increase the value of R_r^c when they are increased. This implies that the HBV infection will grow up in the population when these parameters are increased. On the contrary, γ_2, p, α_0 and γ_3 decrease the value of R_r^c when they are increased. This implies that the disease cannot grow in the population when these parameters are increasing.

The most, sensitive parameter is the transmission coefficient, β . Increasing or decreasing the value of β leads to the increase or decrease of the value of R_r^c with the same proportion since the sensitivity index is equal to one. Therefore, as β increases, many individuals become infected, so HBV transmission increases in the population. Vaccine efficacy p is also sensitive, showing that as p increases R_r^c decreases. This implies that vaccinated individuals have high protection against HBV infection.

Furthermore, when treatment rate, α_0 increases then R_r^c decreases, this implies that many chronic carriers move to the removed class after treatment. Therefore, to minimize HBV transmission in a population, this study recommends that, the combination of vaccination and treatment should be implemented because vaccination reduces the livelihood of an individual to get infected and treatment helps to prevent the spread of disease and its complications.

5. Conclusion

This study is motivated by the work of Nwaokolo et al. (2020) which is an extension of the work of Khan et al. (2016) to concentrate on the effect of immunization and treatment in controlling the spread of HBV with infective travelers. The sensitivity analysis of the model parameters is done to show some of the important parameters that may cause a decrease in the effective reproduction number. Moreover, numerical simulation result shows that, adherence to high-effective immunization and treatment choices ought to be carried out, in other to control HBV transmission with infective travelers. This is in opposition to the outcome acquired by Khan et al. (2016). In light of the discoveries, despite the fact that immunization alone at 80% inclusion has the ability to forestall HBV-related demise around the world, it is alluring to consolidate it close by treatment since it gives the most ideal choice (system) for overseeing HBV cases. Additionally, the outcome from the review proposed that people ought to stick to effective treatment choices if the result of drug resistance (relapse) is to be eliminated during the control of hepatitis B infection.

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