

Modern Method of Replicating Square Matrices

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Abstract

Matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns; it has vast applications in statistics and mathematics in particular and in other works of life in general. They are used to represent, manipulate data and perform operations. The aim of this research is generating modern method of replicating square matrices. The methods and techniques adopted in this research is valid and reliable because of the numerous researches related to this study had used which was indeed outstanding, the matrices can be replicated up to n^2-1 time, the study conclude that for large number of square matrices modern method should be adopted.

Keywords: square matrices, replication, modern method, rows and column.

1 Introduction

Olayiwola *et al.*, (2022) stated that the history of mathematics suggests the discovery of matrices may have started with the existence of magic squares. About 4000 years ago, magic squares were engraved on stones, metals, or paintings. A matrix is a rectangular array of numbers; the horizontal array of factors in the matrix is called rows and the vertical array of factors in the matrix is called the columns. Matrix is an essential step in understanding this fundamental mathematical concept. A matrix is a two-dimensional array of numbers, symbols, or expressions organized in rows and columns. It provides a structured way to represent and manipulate data, equations, and various mathematical concepts. It is widely used in linear algebra, a branch of mathematics that deals with systems of linear equations, vectors, and transformations. Matrices have numerous applications, including solving systems of linear equations, transforming 2D and 3D graphics, data analysis, and more (Kudtarkar & Ansari, 2023). They further noted that Matrices are a versatile tool with broad utility in mathematics, science, engineering, and various other fields. They are a foundation for many advanced mathematical concepts and techniques. This research paper aimed at generating modern method of a set of square matrices. Many research lay more emphasis on the construction of 2×2 and 3×3 square matrices not minding the constructing of 4×4 and larger matrices, several benefits can be derived from making accurate construction of higher square matrices looking at their importance for researchers most especially if one is dealing with larger number of treatments and fewer blocks. For instance, in a plant breeding of partially balanced incomplete block designs which require lesser experimental material where plants (treatments) will be allocated to fewer number of experimental unit (blocks). The aim of this research is modern method of replicating square matrices. The objectives of working with matrices can vary depending on the specific context but for this research there is need to construct a simplest and easy method (modern method) of generating a set of square matrices so that it ease the work of intending researchers on this areas.

2. Related Works

Nair, K.R. (1951) has constructed some two replicate PBIB designs. Recent developments have seen some more efficient designs constructed in (Salihu *et al.*, 2021; Sharma & Garg, 2022) etc in which there is restriction to the number of replications of treatments. Bearing in mind their broad area of applications, there is need to generate a modern method of replicating a set of square matrices so that it ease the work of intending researchers in–these areas.–What differentiate this work with other is that matrices can be generated $n^2 - 1$ times whereby each of the treatments appears in every position of the square matrix. However, in-(Salihu *et al.*, 2021; Sharma & Garg, 2022) matrices are not generated more than $n^{1/2}$ and n^2 times respectively and any attempt to generate more matrices, the resulting matrices will be duplicated, this makes the algorithms for generating matrices inconsistent.

According to the results of researchers who conducted comparable studies showed there are restrictions to the number of replications.

3. Methodology

Let $v = S^2$ ($S = 3, 4$) treatments which are arranged in ‘S’ rows and ‘S’ columns in a square matrix ‘M’ as given below

$$M = \begin{pmatrix} 1 & 2 & \cdot & \cdot & s \\ s + 1 & s + 2 & s + 3 & \dots & 2s \\ 2s + 1 & 2s + 2 & 2s + 3 & \dots & 3s \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (s - 1)s + 1 & (s - 1)s + 2 & (s - 1)s + 3 & \cdot & s^2 \end{pmatrix}$$

From these ‘v’ symbols arranged in ‘s’ rows and ‘s’ columns, where s is a positive integers such that (s takes values 4 only).

From the matrix (M) we can obtain (n-1) more matrices giving us in total a set $S = \{S_{(1)}, S_{(2)}, \dots, S_{(K)}\}$ of $k = n^2$ by following the algorithm below

Step 1: from a given matrix, subsequent matrices (2, 3 and 4) will be obtained by moving the rows of the given matrix one step upwards and taking the first row of the given matrix as the last row.

Step 2: move columns of the given matrix one step towards left taking the first column of the given matrix as the last column to obtain succeeding matrix.

Step 3: repeat step 1 above to obtain matrices 6, 7 and 8.

Step 4: repeat step 2 above to obtain matrix 9.

Step 5: To obtain other matrices follow the procedures above

Step 6: To generate matrices above sixteen, transpose the resulting sixteenth matrix

4. Results and Discussion

There are numerous researchers who worked on the areas of generating square matrices but there are so much limitation attached to their work and this gave rise to modern method of generating a set of square matrices so that intending researchers on this area can have a relief.

The following are the result of the matrices obtained by using above algorithm

Original matrix

$$\text{let } A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

| | | | |
|---|---|---|---|
| replication 1 | replication 2 | replication 3 | replication 4 |
| $\begin{pmatrix} 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ | $\begin{pmatrix} 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$ | $\begin{pmatrix} 13 & 14 & 15 & 16 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$ | $\begin{pmatrix} 2 & 3 & 4 & 1 \\ 6 & 7 & 8 & 5 \\ 10 & 11 & 12 & 9 \\ 14 & 15 & 16 & 13 \end{pmatrix}$ |
| replication 5 | replication 6 | replication 7 | replication 8 |
| $\begin{pmatrix} 6 & 7 & 8 & 5 \\ 10 & 11 & 12 & 9 \\ 14 & 15 & 16 & 13 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ | $\begin{pmatrix} 10 & 11 & 12 & 9 \\ 14 & 15 & 16 & 13 \\ 2 & 3 & 4 & 1 \\ 6 & 7 & 8 & 5 \end{pmatrix}$ | $\begin{pmatrix} 14 & 15 & 16 & 13 \\ 2 & 3 & 4 & 1 \\ 6 & 7 & 8 & 5 \\ 10 & 11 & 12 & 9 \end{pmatrix}$ | $\begin{pmatrix} 3 & 4 & 1 & 2 \\ 7 & 8 & 5 & 6 \\ 11 & 12 & 9 & 10 \\ 15 & 16 & 13 & 14 \end{pmatrix}$ |
| replication 9 | replication 10 | replication 11 | replication 12 |
| $\begin{pmatrix} 7 & 8 & 5 & 6 \\ 11 & 12 & 9 & 10 \\ 15 & 16 & 13 & 14 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ | $\begin{pmatrix} 11 & 12 & 9 & 10 \\ 15 & 16 & 13 & 14 \\ 3 & 4 & 1 & 2 \\ 7 & 8 & 5 & 6 \end{pmatrix}$ | $\begin{pmatrix} 15 & 16 & 13 & 14 \\ 3 & 4 & 1 & 2 \\ 7 & 8 & 5 & 6 \\ 11 & 12 & 9 & 10 \end{pmatrix}$ | $\begin{pmatrix} 4 & 1 & 2 & 3 \\ 8 & 5 & 6 & 7 \\ 12 & 9 & 10 & 11 \\ 16 & 13 & 14 & 15 \end{pmatrix}$ |
| replication 13 | replication 14 | replication 15 | |
| $\begin{pmatrix} 8 & 5 & 6 & 7 \\ 12 & 9 & 10 & 11 \\ 16 & 13 & 14 & 15 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ | $\begin{pmatrix} 12 & 9 & 10 & 11 \\ 16 & 13 & 14 & 15 \\ 4 & 1 & 2 & 3 \\ 8 & 5 & 6 & 7 \end{pmatrix}$ | $\begin{pmatrix} 16 & 13 & 14 & 15 \\ 4 & 1 & 2 & 3 \\ 8 & 5 & 6 & 7 \\ 12 & 9 & 10 & 11 \end{pmatrix}$ | |

5. Conclusion

In this paper, square matrices have been generated $n = 4$, $n^2 = 16$ & $n^2 - 1 = 15$. This show there is restriction to the number of replication of square matrices generated. The study recommends that the modern method of generating a square matrix should be adopted because of its uniqueness.

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